(I)Solve the equations:

(1) $2x + 5 = 0$ (2) $\frac{2}{3}x - 3 = 0$ (3) $2x = 0$

(4) $\frac{1}{3}x^2 - 3 = 0$ (5) $\frac{1}{2}x^2 + 2 = 0$ (6) $\frac{1}{2}x^3 - 2x = 0$

(7) $x^2 - 3x + 2 = 0$ (8) $x^3 - 3x^2 + 2x = 0$ (9) $x^4 - x = 0$

(10) $2^x - 3 = 0$ (11) $2^x - 5^x = 0$ (12) $2^x = \frac{1}{2}3^x$

(13) $4 - 2^x = 1$ (14) $2^{x+1} - 5^x = 0$ (15) $2^{x-1} = \frac{1}{2}3^{x+1}$

(16) $(x - 1)2^x = 0$ (17) $(x + 1)4^x = 3^x$ (18) $2^x + 3^x = 0$

(19) $\ln(x - 1) = 0$ (20) $\log x = 1$ (21) $\ln(x + 2) = 3$

(22) $\log_3 x = -2$ (23) $\log_4(x + 1) = 2$ (24) $x \ln(x - 3) = 0$

➢ Limits

Find the following limits:

(1) $\lim_{x\to 2}(x^2 + 3x)$ (2) $\lim_{x\to 2}\frac{x^4 - 16}{\sqrt{x} - \sqrt{2}}$ (3) $\lim_{x\to 3}\frac{x^3 - 27}{x - 3}$

(4) $\lim_{x\to -2}\frac{x^3 + 8}{x + 2}$ (5) $\lim_{x\to 4}\frac{x^2 - 1}{\sqrt{x} - 1}$ (6) $\lim_{x\to 1}\frac{x^3 - 1}{x - 1}$

(7) $\lim_{x\to -1}\frac{x^5 + 1}{x + 1}$ (8) $\lim_{x\to 1}\frac{x^7 - 1}{x^5 - 1}$ (9) $\lim_{x\to 2}\frac{x^3 - 8}{x^5 - 32}$

(10) $\lim_{x\to 0}\frac{\sin x}{x + 2}$ (11) $\lim_{x\to \pi}\frac{\sin x}{x - 1}$ (12) $\lim_{x\to 0}\frac{x}{x + \tan x}$

(13) $\lim_{x\to 0} x \sin x$ (14) $\lim_{x\to 0}\frac{\sin 3x}{2x}$ (15) $\lim_{x\to \pi}\frac{\sin x}{x - \pi}$

(16) $\lim_{x\to 0}\frac{x^4}{(\sin x)^5}$ (17) $\lim_{x\to 0}\frac{(\sin x)^4}{4x}$ (18) $\lim_{x\to 0}\frac{x - \sin x}{x + \tan x}$

(19) $\lim_{x\to 1}\frac{\log(1 + 9x)}{x}$ (20) $\lim_{x\to 0}\frac{\log(1 + 2x)}{x}$ (21) $\lim_{x\to 0}\frac{\ln(1 + x)}{3x}$

(22) $\lim_{x\to 0}\frac{\log(1 + x)}{\ln(1 + 2x)}$ (23) $\lim_{x\to 0}\frac{2x}{\ln(x + 1)}$ (24) $\lim_{x\to 1}\frac{\ln x}{x - 1}$

(25) $\lim_{x\to 0}\frac{\log(1 + x)}{\sin x}$ (26) $\lim_{x\to 0}\frac{\ln(x + 1)}{x + \tan x}$ (27) $\lim_{x\to 0}\frac{\ln(1 + x)}{x + \cos x}$

(28) $\lim_{x\to 0}\frac{2^x - 1}{x}$ (29) $\lim_{x\to 0}\frac{3^x - 1}{4x}$ (30) $\lim_{x\to 0}\frac{2^x - 3^x}{x}$

(31) $\lim_{x\to 0}\frac{2^x - 3^x}{4^x - 5^x}$ (32) $\lim_{x\to 0}\frac{x^2}{2^x - 1}$ (33) $\lim_{x\to 0}\frac{\sin x}{2^x - 1}$

(34) $\lim_{x\to \infty}\frac{x + 2}{x^2 - 2x + 1}$ (35) $\lim_{x\to \infty}(x^3 + 2)$ (36) $\lim_{x\to \infty}\frac{x^3 + 2}{x^2 - x}$

(37) $\lim_{x\to \infty}\frac{x + 1}{\sqrt{x^2 + 1}}$ (38) $\lim_{x\to \infty}\sqrt{\frac{x^4 + 1}{x^4 + 3}}$ (39) $\lim_{x\to \infty}\frac{4 - 3x^2}{x^2 + 2x}$

➢ Differentiation

(I)Find the first derivative of the following functions:

(1) $f(x) = x^3 + 4x^2 + 5$ (2) $f(x) = 3^x + 4^x + 2$ (3) $f(x) = \log x + \ln x$

(4) $f(x) = x^2.3^x + \log_4 x$ (5) $f(x) = \sqrt{x} + x^3.4^x$ (6) $f(x) = \dfrac{x+1}{x^2+5}$

(7) $f(x) = 2\sin x + 3\tan x$ (8) $f(x) = \sec x + 2\cos x$ (9) $f(x) = 2\sin 3x + \tan x^3$

(10) $f(x) = 3\sin(2x+1)$ (11) $f(x) = \cot x.\cos 2x$ (12) $f(x) = 4^x.\tan x^3$

(13) $f(x) = x^3 + \dfrac{\sin 2x}{x+2}$ (14) $f(x) = x^3.3^x.\sin x$ (15) $f(x) = \dfrac{\tan 3x}{x} + \ln x$

(16) $f(x) = 3x + \ln(x+3)$ (17) $f(x) = 2^{\sin x} + \tan x$ (18) $f(x) = \log(1+\sin x)$

(19) $f(x) = 8^x + 3\sin 2x$ (20) $f(x) = \sin x + \cos 3x$ (21) $f(x) = \tan x + 2\cos x$

(22) $f(x) = x + \sin^{-3} x$ (23) $f(x) = 3^x.\sin\sqrt{x}$ (24) $f(x) = \ln x.\cos^{-4} 2x$

(25) $f(x) = x.\tanh x^3$ (26) $f(x) = 4^x.\cos x^3$ (27) $f(x) = \sin(x^2+1)$

(28) $f(x) = (5x+x^3)^4$ (29) $f(x) = (x^2+\sin x)^5$ (30) $f(x) = (3^x+\sin x)^{-4}$

(31) $f(x) = (\tan x+\ln x)^6$ (32) $f(x) = \sin^{-3} x + \sin x$ (33) $f(x) = \sin^4 x + \sin x^4$

(34) $f(x) = (x+x^2)^8 + \cos x$ (35) $f(x) = \tan^3 x + \tan x^3$ (36) $f(x) = \sqrt{4x + \sin 3x}$

(II)Find y` from the following :

(1) $y = x^3 + \sqrt[3]{x+2}$ (2) $y = x^3.\sin x.\tan x$ (3) $y = x^2\cot x + 8$

(4) $y = \sqrt{\sin 2x} + \sqrt{x}$ (5) $y = 3x + \sec 2x$ (6) $y = \sec x + \log x$

(7) $y = \cos x + \cos x^{-1}$ (8) $y = \sqrt{x + \sqrt{x+2}}$ (9) $y = e^{2x} + e^{\sin x}$

(III)Find y` and y`` from the following :

(1) $y = 8 + 3^x + \log x$ (2) $y = 3x + \ln(2x+3)$ (3) $y = \sin 3x + \dfrac{3}{x+1}$

(4) $y = x^{-2} + (x+2)^{-3}$ (5) $y = x.\cos x$ (6) $y = x^2 + \tan^{-2} x$

(IV) Find y` at the given points in the following :

(1) $y = x^3 + x\sin x$, at $x = 0$ (2) $y = 3^x.\ln(2x-1)$, at $x = 1$

(3) $y = \cos x + \ln(x+1)$, at $x = 0$ (4) $y = x^4 + 3x + 23$, at $x = 2$

➢ Applications of Differentiation

(I)Find the extrema of the following functions:

(1) $f(x) = 2x + 1$
(2) $f(x) = 3^x + 2$
(3) $f(x) = \log x$
(4) $f(x) = \ln x$
(5) $f(x) = \frac{x}{2} + \frac{2}{x}$
(6) $f(x) = \frac{3}{4} + \frac{1}{1+x^2}$
(7) $f(x) = 2x^2 - 8x + 1$
(8) $f(x) = 4x - x^2$
(9) $f(x) = x^3 - 12x$
(10) $f(x) = 3 + (x-2)^4$
(11) $f(x) = 2x^3 - 6x$
(12) $f(x) = x^3 - 3x^2 - 9x$
(13) $f(x) = x + \cos x$
(14) $f(x) = 3x - (x-2)^3$
(15) $f(x) = \sqrt[3]{(x-1)^2}$
(16) $f(x) = 1 - \sqrt[3]{x}$
(17) $f(x) = (x^2-1)^2$
(18) $f(x) = e^{2x} + 2e^{-x}$

(III)Using L'Hopital rule, find the following limits:

(1) $\lim_{x\to 0}(x^2 + x + 3)$
(2) $\lim_{x\to 4}\frac{x^2-1}{\sqrt{x}-1}$
(3) $\lim_{x\to 1}\frac{x^3-1}{x-1}$
(4) $\lim_{x\to 0}\frac{\sin x}{x+2}$
(5) $\lim_{x\to \pi}(\sin x - \cos x)$
(6) $\lim_{x\to 0}\frac{x}{x+\tan x}$
(7) $\lim_{x\to 0}(x\sin x - \cos x)$
(8) $\lim_{x\to 0}\frac{\sin 3x}{x}$
(9) $\lim_{x\to 0}(\frac{3x}{\sin x} + 2x)$
(10) $\lim_{x\to 0}\frac{x^2}{\sin 2x}$
(11) $\lim_{x\to 0}\frac{2x}{\tan 3x}$
(12) $\lim_{x\to 0}\frac{\sin 2x}{x^3+2x}$
(13) $\lim_{x\to \pi}\frac{\sin x}{\pi - x}$
(14) $\lim_{x\to \pi}\frac{x-\pi}{1+\cos x}$
(15) $\lim_{x\to \pi}\frac{1+\cos x}{\pi - x}$
(16) $\lim_{x\to 1}\frac{\log(1+9x)}{x}$
(17) $\lim_{x\to 0}\frac{\log(1+2x)}{x}$
(18) $\lim_{x\to 0}\frac{2x}{\ln(1+3x)}$
(19) $\lim_{x\to 1}\frac{2^x-1}{x}$
(20) $\lim_{x\to 0}\frac{x}{1-3^x}$
(21) $\lim_{x\to 0}\frac{3^x-2^x}{4^x-5^x}$
(22) $\lim_{x\to \infty}\frac{x^2-2x}{2^x+x}$
(23) $\lim_{x\to 0}\frac{2^x-7^x}{5^x-3^x}$
(24) $\lim_{x\to \infty}\frac{x^2+1}{x+3^x}$
(25) $\lim_{x\to \infty}(2x-3)$
(26) $\lim_{x\to \infty}\frac{2x+1}{x+3}$
(27) $\lim_{x\to \infty}\frac{x^2+3}{x^3+2x-5}$
(28) $\lim_{x\to \pi}(\csc x - \cot x)$
(29) $\lim_{x\to \pi}(\sec\frac{x}{2} - \tan\frac{x}{2})$
(30) $\lim_{x\to \infty}(\frac{1}{x} - \frac{1}{x^2})$

(IV)Write the Taylor's expansion of each of the following functions:

(a) $f(x) = 2 + \frac{1}{x^2}$ at x = 1
(b) $f(x) = x^2 + 2^x$ at x = 2

(c) $f(x) = \ln x$ at x = 1 (d) $f(x) = \ln(x-2)$ at x = 3

(e) $f(x) = \sin x$ at x = π/2 (f) $f(x) = \cos x$ at x = π

(V)Write the Maclaurin's expansion of each of the following functions:

(a) $f(x) = \frac{1}{x-1}$ (b) $f(x) = \cos 2x$ (c) $f(x) = \sin 3x$

(d) $f(x) = \ln(x+1)$ (e) $f(x) = x + 3^x$ (f) $f(x) = x^3 + 2x$

(g) $f(x) = \sin x^3$ (h) $f(x) = \cos x^2$ (i) $f(x) = \tan x$

(k) $f(x) = \sqrt{1+x}$ (l) $f(x) = \sqrt{1+x^2}$ (m) $f(x) = \frac{1}{1-x^2}$

(VI)A piece of wire of length 10 meter. At a point P on it, it is bent into L-shape.

Find the distance of P from each end of the wire such that the distance between the ends of the new shape is minimum.

(IV)A piece of wire of length 10 meter is divided into two pieces. One of the pieces is bent into shape of a circle and the other into shape of a square. Find the point of division such that the area enclosed by the two pieces is minimum.

➢ Indefinite Integrals

Find the following integrals:

(1) $\int (x^3 + 2x^2 - 1)\,dx$ (2) $\int (2x^4 - 3x^2 + 3)\,dx$ (3) $\int (x^{-3} + \sqrt{x} + \frac{3}{2})\,dx$

(4) $\int (x^{-2} + \frac{2}{x^3} + \frac{2}{3})\,dx$ (5) $\int (3^x + 5^x + x)\,dx$ (6) $\int (4^x - 2^x + x^3)\,dx$

(7) $\int [(\frac{3}{5})^x + \frac{2^x}{3^x}]\,dx$ (8) $\int [(\frac{2}{3})^x + \frac{4^x}{3^x}]\,dx$ (9) $\int [x^4 + \frac{1}{3^x}]\,dx$

(10) $\int [\frac{1}{4^x} - x^{-3}]\,dx$ (11) $\int [2 - x^3]^2\,dx$ (12) $\int [3 - x^2]^2\,dx$

(13) $\int [1 + 3^x]^2\,dx$ (14) $\int [2 - 3^x]^2\,dx$ (15) $\int [2^x + 3^x]^2\,dx$

(16) $\int [4^x - 3^x]^2\,dx$ (17) $\int 2x.3^{x^2}\,dx$ (18) $\int 3x^2.4^{x^3}\,dx$

(19) $\int \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx$ (20) $\int \frac{1}{x^2} e^{\frac{1}{x}} dx$ (21) $\int \cos x\,.e^{\sin x}\,dx$

(22) $\int [3x + \cos 2x]\,dx$ (23) $\int [2x - \sin 3x]\,dx$ (24) $\int (\frac{1}{x^2} + \tan 2x)dx$

(25) $\int [3^{2x} - \cos 3x]\,dx$ (26) $\int [2^{3x} + 3\sin 2x]\,dx$ (27) $\int [\cos x + \sin x]\,dx$

(28) $\int [3^x - \cos x]\,dx$ (29) $\int (2 - \sin 2x)\,dx$ (30) $\int [\tan x + \tan 2x]\,dx$

➢ Matrices

(1)If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}$

Find A + B, A + B + C, A + 2B + 3C, A`, C`, AB, AC, ABC, |A|, |B|, |AB|, |BA|.

(2)If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 3 \\ 1 & 2 \\ 3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 4 \\ -1 & 0 & 2 \end{bmatrix}$

Find if possible : A + B, $A + B^t$, A + C, A.B , A.A, A.A`, C.C`, A.C , B.C , |A| , |C| , |A.B|.

(3)Find the inverse of each matrix, if exists :

$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 1 \\ 9 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 4 \\ -1 & 0 & 2 \end{bmatrix}$

$E = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & 7 \end{bmatrix}$, $F = \begin{bmatrix} 2 & 1 \\ 0 & 9 \\ 2 & 1 \end{bmatrix}$, $G = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, $H = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 4 \\ 2 & 5 & 4 \end{bmatrix}$

(4)Find the eigenvalues and eigenvectors of the following matrices, if possible:

$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -2 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 2 & 5 \end{bmatrix}$

(5)Find the eigenvalues and eigenvectors of the inverse of the matrix A given in (4).

Show the relation between the eigenvalues of A and its inverse.

(6)Find the eigenvalues and eigenvectors of the following matrices, if possible:

$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

$E = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{bmatrix}$, $F = \begin{bmatrix} 3 & -2 & -5 \\ 4 & -1 & -5 \\ -2 & -1 & -3 \end{bmatrix}$, $G = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

➢ Linear Systems

(1)Write the augmented matrix of each of the following linear systems and determine the type of solution.

(a) $x + y = 5$ $2x - y = 1$	(b) $x + y = 5$ $2x - y = 1$ $-2x - 2y = -10$	(c) $x + y = 5$ $2x - y = 1$ $x + 2y = 8$	(d) $x + y - z = 4$ $2x - y + 3z = 5$ $-x - y + z = -4$
(e) $x + y = 4$ $y - x = 3$	(f) $x + 2y = 4$ $2x - y = 0$ $3x + y = -1$	(g) $x + y = 3$ $2x - z = 4$ $2y + z = 8$	(h) $x + 2y - z = 1$ $x - y + 3z = 3$ $2x + y + 2z = 5$

(2)Solve the following linear systems by Gauss, Crammer, inverse methods, if possible:

(a) $x + y = 3$ $3x - y = 1$	(b) $2x + y = 6$ $3x - y = 4$	(c) $x + y + z = 5$ $2x - y + z = 2$ $2x + 2y - z = 4$
(d) $x + y - z = 3$ $x - y + 2z = 5$ $2x + 2y - 2z = 6$	(e) $2x + y + 2z = 8$ $x - y + z = 1$ $x + y + 2z = 7$	(g) $x_1 + x_2 - x_3 + x_4 = 4$ $2x_1 + x_2 + x_3 - x_4 = 3$ $x_1 - x_2 + 2x_3 + x_4 = 6$ $-x_1 + x_2 + x_3 - x_4 = 0$

(3)Solve the following linear system:

$x_1 + x_2 + x_3 + x_4 = 10, \quad x_1 - x_2 + 2x_3 - x_4 = 1$

$2x_1 + x_2 - x_3 + x_4 = 5, \quad x_1 + x_2 + 2x_3 - 2x_4 = 1$

➢ Binomial Expansion

(1)Expand each of the following :

(a) $\sqrt{1+3x}$ (b) $\sqrt{4-3x}$ (c) $\sqrt[3]{1-2x^2}$ (d) $\frac{1}{\sqrt{4-2x}}$

(e) $\frac{1}{x^2-4x+3}$ (f) $\frac{3}{2-3x}$ (g) $\frac{4}{\sqrt[3]{8-3x}}$ (h) $\frac{1}{\sqrt{4-x^2}}$

➢ Complex Numbers

(1) If $z_1 = 2 + 3i$ and $z_2 = 3 - i$. Find

(i) $z_1 + z_2$ (ii) $z_1 \cdot z_2$ (iii) $2z_1 - 3z_2$ (iv) z_1 / z_2

(2)Put the following complex numbers in polar form:

(i)z = 4 + 4i (ii)z = – 4 (iii) z = 2i (iv) z = $-1+i$

(3) If $z_1 = -1 + i$, $z_2 = -2i$. Find $z_1 + z_2$, $z_1 \cdot z_2$, $\frac{z_1}{z_2}$, $(z_1)^8$, $\sqrt[4]{z_2}$

(4) If $z_1 = -1 - i$, $z_2 = 2 + 2i$. Find $z_1 + z_2$, $z_1 \cdot z_2$, $\frac{z_2}{z_1}$, $\frac{\sqrt[4]{z_1}}{(z_2)^6}$, $\frac{(z_1)^4}{(z_2)^6}-$

➢ Partial Fractions

Resolve into partial fractions:

(1) $\frac{x}{x^2-5x+6}$ (2) $\frac{3}{x^2-5x}$ (3) $\frac{x+2}{x^2-3x-4}$ (4) $\frac{2x+3}{x^3-5x^2+6x}$

(5) $\frac{3}{x^2-4x+4}$ (6) $\frac{x+1}{x^2-6x+9}$ (7) $\frac{x-3}{x^2-2x+1}$ (8) $\frac{x^2+3}{x^3-4x^2+4x}$